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Supplementary information

Supersonic impact resilience of nanoarchitected carbon

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Supplementary Information: *Supersonic Impact Resilience of Nanoarchitected Carbon*

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I. SiO_2-Si calibration impacts and SiO_2 consolidation

The impact response of the SiO_2 spheres onto Si was characterized by distinct rebound and shattering regimes. Representative frames from a rebound-regime impact are shown in Extended Data Fig. 4a, corresponding to impact and rebound velocities of $v_0 = 514$ m/s and $v_r = 339$ m/s, respectively. This regime extended to impact velocities of ~650 m/s, around which the shattering regime emerged. A representative impact in this regime is presented in Extended Data Fig. 4b, corresponding to an impact velocity of 646 m/s. The last frames in this sequence show catastrophic failure of the particle which disintegrated into several pieces as the one shown in Extended Data Fig. 4d.

Using the velocities obtained from the camera frames, and assuming a density $\delta = 1,850$ kg/m^3 for SiO_2 (microParticles GmbH), a radius $r = 14$ µm, and a mass $m = \frac{4}{3}\delta\pi r^3$ for all particles, the impact energy $W_0 = \frac{1}{2}mv_0^2$ and rebound energy $W_r = \frac{1}{2}mv_r^2$ for each impact was calculated. Additionally, the dissipated or inelastic energy was computed as the difference between the impact and rebound energies, $W_{i,SiO_2} = W_0 - W_r$. Normalizing the rebound and inelastic energies by the impact energy and plotting those values as a function of the impact energy (Extended Data Fig. 5a) shows a non-linear increase in dissipation with increasing impact energy.

To understand this anomalous behavior of the SiO_2 particles under high-velocity impact, we refer to the continuum plasticity model proposed by Schill et al.[1]. By applying this model, we assume that inelastic energy is primarily accounted for by densification (i.e., consolidation) of the SiO_2 spheres due to the high pressures associated with supersonic impact, and thereby neglect contributions of heat loss or damage in the Si substrate (no damage was visible in post-mortem SEM micrographs). The consolidation energy density in SiO_2 as a function of J_p, the permanent volumetric deformation factor (i.e., the resulting fraction of the original volume after impact), can be obtained by integrating the consolidation relation presented in this model

$$p_c = p_0 + \frac{A}{\alpha}\left(1 - J_p^{-\alpha}\right), \tag{S1}$$

where p_c is the consolidation pressure (i.e., maximum pressure attained) and A, p_0, and α are fit parameters provided by the model[1]. Taking this energy density and multiplying it by the particle volume provides an inelastic energy estimate assuming an on-average particle consolidation J_p, which is plotted in Extended Data Fig. 5b. Matching the maximum inelastic energy observed experimentally to the energies predicted by the model, corresponding to an impact velocity of v_0 = 696 m/s, indicates that an on-average volume reduction of up to ~9% (for $J_p \approx 0.91$) occurred for particles that did not shatter. Impact velocities beyond the shatter limit are assumed to have caused an on-average consolidation pressure higher than ~4.3 GPa, as shown in Extended Data Fig. 5c, which could lead to unstable behavior at a defect to initiate fracture. This model validates our approximation that internal processes of the SiO_2 particles can indeed account for the dissipated energy in these SiO_2-Si impacts.

Since the energy loss associated with inelastic processes in SiO_2 is not negligible, we fit the experimental data to a second-order polynomial of the form $W_{i,SiO_2} = C_1 W_r^2 + C_2 W_r + C_3$ (with $C_1 = 5.94 \times 10^6$, $C_2 = -0.126$, and $C_3 = 1.34 \times 10^{-9}$) to provide a function that related the rebound and inelastic energies, as shown in Extended Data Fig. 6. The motivation behind this fit was to estimate the dissipation energy of the SiO_2 particles to decouple this mechanism from other dissipation mechanisms in our experiments. In particular, this function was later used to decouple the inelastic contributions from the SiO_2 projectiles and the nano-architected carbon.

II. Shock propagation analysis

Shock wave propagation in foams has been observed to occur at critical velocities well below the elastic wave speeds[2] and is characterized by a compaction front across which stress, strain, and particle velocities are discontinuous. Just as in stochastic foams, lattice materials have been shown to support shock propagation both at the macro- and micro-scales[3,4]. Although a thorough analysis of the shock physics mechanisms in our samples was beyond the scope of this study, here we analyze the general propagation of shocks under an impact stimulus such as in our experiments. It is important to note that although a compaction or densification regime was not observed in our quasi-static compression experiments due to portions of the sample being ejected upon brittle failure events, we are assuming the confinement provided by material surrounding the craters to enable a compaction regime—as evidenced by compacted debris at the bottom of the craters (Extended Data Fig. 3). Additionally, due to the cylindrical crater morphology (and for simplicity), we assume a one-dimensional shock analysis to be valid.

Following Barnes et al.[2], we can express conservation of mass, momentum, and energy across the shock in the Lagrangian reference frame as

$$\rho_0 \dot{s}\, [\![-\rho^{-1}]\!] = [\![v]\!], \tag{S2}$$

$$\rho_0 \dot{s}\, [\![v]\!] = [\![\sigma]\!], \tag{S3}$$

$$\rho_0 \dot{s}\, [\![U + \frac{1}{2}v^2]\!] = [\![\sigma v]\!], \tag{S4}$$

respectively, where $[\![\beta]\!] = \beta^+ - \beta^-$ corresponds to the jump operator defining the difference between a given β parameter's value just behind (i.e., the final value) and just ahead (i.e., the initial value) the shock front (see Extended Data Fig. 7). Here, ρ_0 is the initial density of the material, $\dot{s}$ denotes the velocity of the shock front, ρ is the mass density, v is the particle velocity, σ is the nominal stress, and U is the strain energy density.

Assuming transverse strain to be negligible, we have

$$\begin{aligned}\rho_0\, [\![\rho^{-1}]\!] &= \frac{\rho_0}{\rho^+} - \frac{\rho_0}{\rho^-} \\ &= (1-\varepsilon^+) - (1-\varepsilon^-) \\ &= \varepsilon^- - \varepsilon^+,\end{aligned}$$

so mass conservation (Eq. S2) can be expressed as

$$\dot{s}\, [\![\varepsilon]\!] = [\![v]\!]. \tag{S5}$$

Using Eq. S5, we can express momentum conservation (Eq. S3) as

$$\rho_0\, [\![v]\!]^2 = [\![\sigma]\!]\, [\![\varepsilon]\!],$$

and taking the region ahead of the shock to have $\varepsilon^- \approx 0$ yields

$$\sigma^+ = \sigma^- + \frac{\rho_0 (v^+ - v^-)^2}{\varepsilon^+}, \tag{S6}$$

where it has been shown that $\sigma^- \approx \sigma_Y$ in foams[2], i.e., the stress ahead of the shock is approximately equal to the quasi-static collapse stress. Since the v^+–$\dot{s}$ Hugoniot for foams exhibits a linear trend of the form[2]

$$\dot{s} = A + Bv^+, \tag{S7}$$

where A and B are fit parameters and v^+ is the particle velocity behind the shock, substituting this into Eq. S5 yields a useful expression for the strain behind the shock

$$\varepsilon^+ = \frac{(v^+ - v^-)}{A + Bv^+}. \tag{S8}$$

Recognizing that

$$[\![v^2]\!] = [\![v]\!]\, (v^+ + v^-),$$

allows energy conservation (Eq. S4) to be expressed as

$$\rho_0(U^+ - U^-) = \frac{1}{2}(\sigma^+ + \sigma^-)(\varepsilon^+ - \varepsilon^-). \tag{S9}$$

To obtain an estimate of the energy expended across the shock, substituting Eqs. S6 and S8 into Eq. S9, and approximating $U^- \approx 0$, $v^- \approx 0$, $\sigma^- \approx \sigma_y$, and $v^+ \approx v_0$ (where v_0 is the impactor velocity) we have that the expended energy per unit mass is

$$U^+ = \frac{1}{2}v_0^2 + \frac{v_0\,\sigma_y}{\rho_0(A + Bv_0)}. \tag{S10}$$

This expression provides the reasoning behind the inelastic energy decomposition as presented in the main text, i.e., $W_i^* = \frac{1}{2}v_0^2 + W_d^*$, for the nano-architected lattice materials. In particular, it hints to the strength-to-density ratio as the parameter that could maximize energy dissipation in these materials.

TARGET			PROJECTILE					
Material	Thickness	Density	Material	Diameter	Mass	Impact Velocity	Specific Dissipation Energy	Ref.
PMMA	4–6 mm	1190 kg/m^3	Steel	7.8 mm	2.05 g	86–639 m/s	0.034–0.23 MJ/kg	[5]
Polystyrene	75–290 nm	1053 kg/m^3	Silica	3.7 µm	0.05 ng	350–800 m/s	0.4–2.8 MJ/kg	[6]
Aluminum (2024)	1.27 mm	2780 kg/m^3	Steel	6.4–12.7 mm	1–8.4 g	152–869 m/s	0.11–0.56 MJ/kg	[7]
Aluminum (1100–H12)	1 mm	2700 kg/m^3	Steel (EN-24)	19 mm	47 g	92–115 m/s	0.138–0.150 MJ/kg	[8]
Stainless Steel (304)	0.4 mm	7800 kg/m^3	Steel	8 mm	2 g	177–592 m/s	0.091–0.272 MJ/kg	[9]
Stainless Steel (304)	3 mm	7800 kg/m^3	1020 carbon steel	12.5 mm	8.4 g	485–993 m/s	0.198–0.684 MJ/kg	[10]
Spectra900 (polyethylene/vinylester)	6.7 mm	1008 kg/m^3	Tungsten carbide	12.7 mm	16 g	212–365 m/s	0.406–0.437 MJ/kg	[11]
Kevlar/phenolic-polyvinylbutyral	7 mm	1355 kg/m^3	Tungsten carbide	12.7 mm	16 g	296–422 m/s	0.457–0.546 MJ/kg	[11]
Multi-layer graphene	10–100 nm	2200 kg/m^3	Silica	3.7 um	0.05 ng	600–900 m/s	1.10–1.26 MJ/kg	[12]

Supplementary Table 1 | References for spherical projectile ballistic experiments on other materials used for comparison in the main text and Fig. 3c. The density for composite materials was obtained using the rule of mixtures. In all cases, the participation mass (for computing the specific dissipation energy) was taken to be the footprint area of the spherical projectile multiplied by the material thickness.

**Supplementary Table 2 | ** LIPIT data for nano-architected carbon impact. All projectiles were 14 µm-diameter SiO_2 particles with a density of 1850 kg/m^3. Data that was not collected (due to experimental limitations or particle shatter) is indicated as a dash.

Relative Density	Substrate ID	Sample ID	Shot Number	Impact Velocity [m/s]	Rebound Velocity [m/s]	Restitution Coefficient	Crater Volume [um^3]
14	20	1	1	502	162	0.32	3242
14	20	2	1	511	181	0.35	2667
14	20	3	1	482	159	0.33	3374
14	20	4	1	491	182	0.37	3183
14	22	1	1	249	24	0.10	–
14	22	2	1	251	24	0.10	–
14	22	3	1	246	20	0.08	–
14	22	4	1	241	94	0.39	–
14	24	1	1	38	13	0.34	0
14	24	2	2	38	13	0.34	0
14	24	3	2	44	21	0.48	0
14	24	4	1	44	18	0.40	0
14	21	1	1	963	–	–	–
14	21	3	1	808	–	–	–
14	17	1	1	749	296	0.39	3093
14	17	2	1	760	279	0.37	3522
14	17	3	1	699	256	0.37	3563
14	19	1	1	608	253	0.42	2949
14	19	2	1	757	263	0.35	3026
14	19	3	1	545	235	0.43	3742
23	15	1	1	358	20	0.06	1379
23	15	2	1	516	0	0.00	1821
23	23	1	1	238	50	0.21	1037
23	23	2	1	289	46	0.16	1145
23	23	3	1	255	48	0.19	1361
23	23	4	1	265	45	0.17	–
23	26	2	2	44	19	0.43	0
23	25	1	1	31	20	0.63	0
23	25	2	1	44	23	0.52	0
23	25	3	1	41	11	0.27	0
23	21	2	1	906	–	–	–
23	27	1	1	820	26	0.03	2520
23	27	2	1	821	0	0.00	2781
23	27	3	1	675	0	0.00	1826
23	27	4	1	687	66	0.10	1955
23	16	1	1	676	0	0.00	2130
23	16	2	1	757	0	0.00	3430

Supplementary Table 3 | LIPIT data for impact on Si substrates. All projectiles were 14 μm-diameter SiO_2 particles with a density of 1850 kg/m^3. Missing rebound velocities correspond to particle shatter.

Impact Velocity [m/s]	Rebound Velocity [m/s]	Restitution Coefficient	Impact Velocity [m/s]	Rebound Velocity [m/s]	Restitution Coefficient
494	344	0.70	675	–	–
515	339	0.66	695	–	–
498	342	0.69	643	–	–
245	221	0.90	646	–	–
257	227	0.88	770	–	–
252	223	0.89	667	–	–
130	121	0.94	787	–	–
38	36	0.94	975	–	–
43	40	0.94	696	382	0.55
36	34	0.93	844	–	–
39	36	0.93	806	–	–
636	401	0.63	718	–	–
493	351	0.71	616	375	0.61
656	390	0.59	552	352	0.64
644	395	0.61	489	351	0.72
862	–	–	362	294	0.81
824	–	–	416	318	0.76
828	–	–	301	252	0.84
842	–	–	270	234	0.87
730	–	–	224	200	0.89
830	–	–	94	83	0.88
1123	–	–	164	151	0.92
1073	–	–	66	61	0.93
823	–	–	35	32	0.93

Supplementary Video Captions

Supplementary Video 1.
LIPIT experiment of a 14 μm-diameter SiO_2 microparticle impacting a nano-architected tetrakaidecahedron carbon material ($\overline{\rho} \approx 23\%$) at $v_0 = 44$ m/s and elastically rebounding at an angle away from the microscope objective. No damage was observed on the sample after this impact.

Supplementary Video 2.
LIPIT experiment of a 14 μm-diameter SiO_2 microparticle impacting a nano-architected tetrakaidecahedron carbon material ($\overline{\rho} \approx 23\%$) at $v_0 = 238$ m/s, causing cratering and particle rebound at $v_r = 50$ m/s.

Supplementary Video 3.
LIPIT experiment of a 14 μm-diameter SiO_2 microparticle impacting a nano-architected tetrakaidecahedron carbon material ($\overline{\rho} \approx 23\%$) at $v_0 = 676$ m/s, causing cratering and particle capture.

Supplementary Video 4.
LIPIT experiment of a 14 μm-diameter SiO_2 microparticle impacting a thick Si substrate at $v_0 = 514$ m/s and rebounding at $v_r = 39$ m/s.

Supplementary Video 5.
LIPIT experiment of a 14 μm-diameter SiO_2 microparticle impacting a thick Si substrate at $v_0 = 646$ m/s and subsequent shatter.

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